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Stress2D - Deep Beam Application

Let us consider a typical deep beam problem, shown in the picture below.

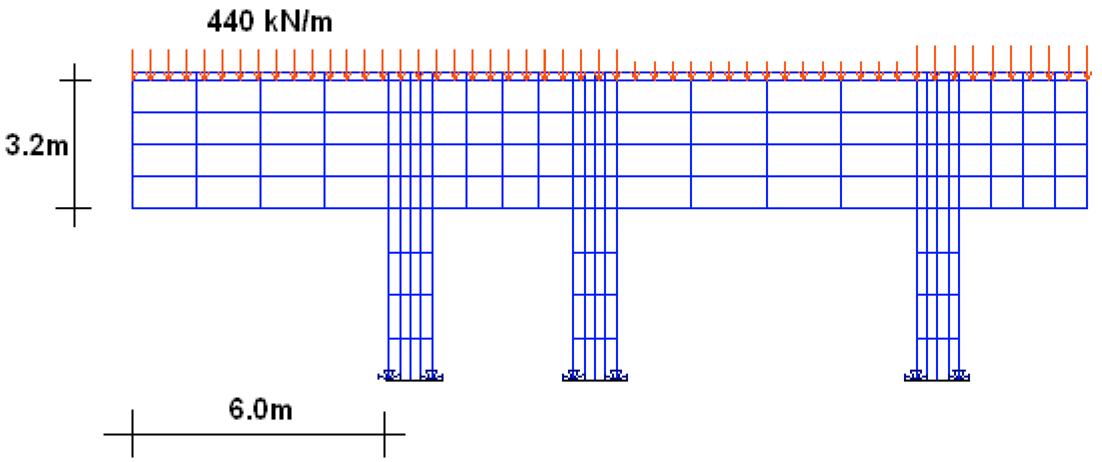

Fig 1. Deep Beam

The cantilever on the left has a section depth of 3.2m, which is half of the 6.0m cantilever span. The section geometry indicates that the cantilever cannot be analysed as an ordinary beam, since the shear deformation will govern the distribution of the internal force and stresses, rather than pure bending as assumed in ordinary beams. In a deep beam, plane sections do not remain plane immediately after the load is applied, and therefore the "beam bending" theory cannot be used. This type of structure requires the use of Finite Element software, such as Stress2D.

This is in accordance with AS 3600, clause 12.1.3.1, where it is required to perform an analysis on deep beams in accordance with acceptable principles of mechanics, as explained in the paragraph above.

In the figure below (Fig. 2) the deformed shape is shown. The deformed shape clearly indicates that the deformations are governed by shear, rather than bending, as it would be the case in an ordinary beam.

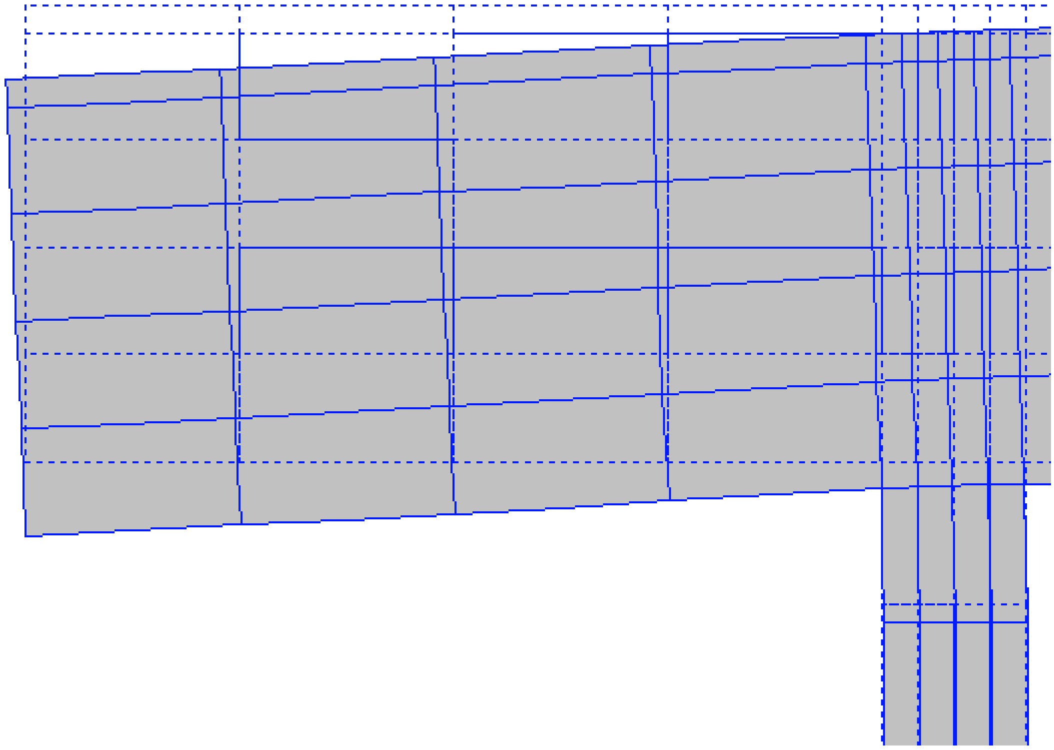

Fig. 2, Deformed Shape

In the figure below (Fig. 3) the horizontal stresses Sigma X are shown at a section near the column. The stress distribution (shown below) is not linear, which is typical for a deep beam.

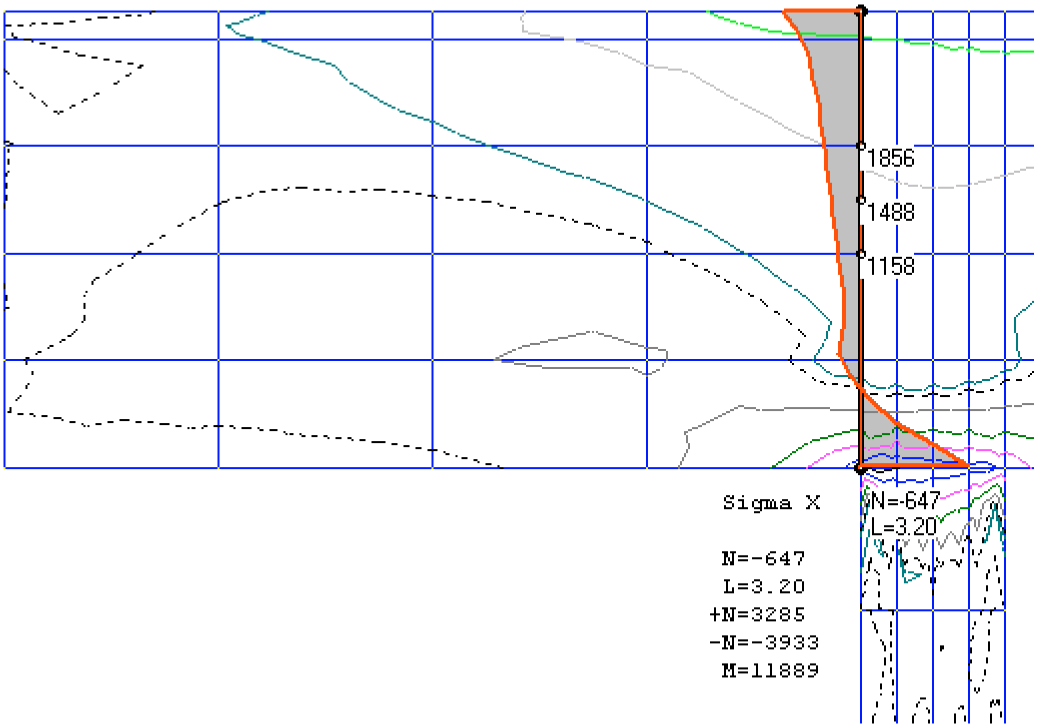

Fig. 3, Stress Distribution

The total section bending moment M (Fig 3), is evaluated by integrating the horizontal stresses

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Sigma X along the section height. This moment may be larger than the moment evaluated by beam theory, which assumes a linear stress distribution. The non-liner stress distribution, typical for deep beams, will generate larger moments in comparison with the linear stress distribution.

If we are to design the horizontal reinforcement for this deep beam, we need to consider the tensile stresses along the vertical section at the column. For example, we can consider a point at the centre of the section where the tensile stresses are 1,488 kPa (Fig. 3). If the section thickness is 0.190m, then the total tensile force over 1m height is:

N = 1,488 x 0.190 x 1.00 = 283 kN tensile force over 1m height

If we ignore the tensile strength of the concrete, then we need to provide a sufficient number of horizontal reinforcing bars to limit the stress in the reinforcing steel to 1/4 of the ultimate steel strength.

If the steel grade is 500 MPa, the maximum stress in the steel will be 125 MPa. This level of stress will provide adequate strength, and it will limit the cracks in the concrete.

Steel Area Required = 283 / 125,000 = 2,264 mm^2

If we use 16mm bars (cross sectional area of $200mm^2$ per bar), then we will need 11 or 12 N16 bars per 1m height. Probably, 6 bars in 2 layers.

Alternatively, we may use tables 8.6.1 (A) and (B) from AS 3600, to determine the maximum stress in the steel bars, which will limit the cracks. For instance, for N16 bars we can use stress of up to 280 MPa if the spacing is limited to 150mm.

The above procedure needs to be repeated for the horizontal section (Sigma Y), and for the shear stresses Tau XY (see Fig. 4) Usually, the shear stress can be taken by the concrete, however if this is not the case then extra bars are required for the shear stresses in both directions.

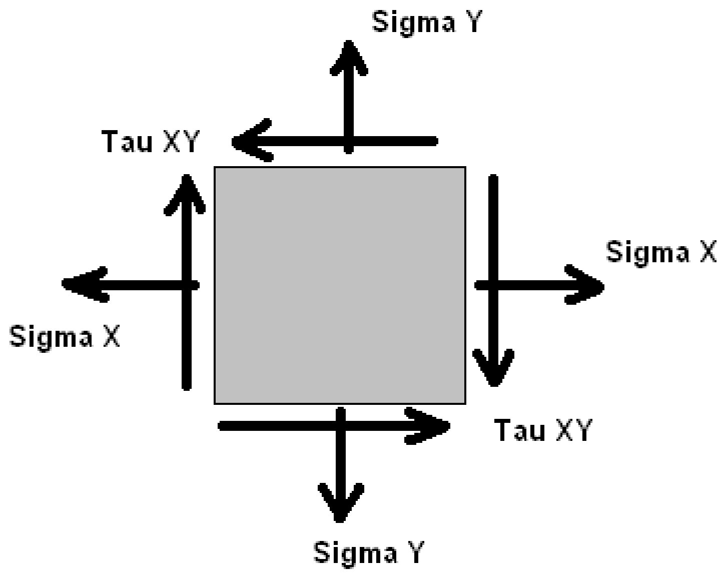

Fig 4, Stresses X-Y

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Another alternative is to assume that the concrete can take some tension. According to AS 3600, clause 6.1.13, the principal tensile strength for concrete grade 32 MPa is 0.4*Sqrt(32) = 2.3 MPa. Based on this assumption, we may say that no horizontal reinforcement is required at the mid-point of the section, since the horizontal stress of 1.5 MPa is below the principal tensile strength of 2.3 MPa. However, we recommend that the tensile strength of the concrete is ignored in the design of deep beams as a much safer alternative.

We alsoe have to check if the compressive stresses are below the concrete compression strength. The tensile stresses have to be taken by the reinforcement in the direction of the principle stresses in tension, limiting the steel stresses to 25% of the yield stress. Then the inclined reinforcement may be split in the X and Y directions, using vector geometry principles.

Note that the above procedure does not consider any global or local buckling.

The most accurate approach would be to consider the principal stresses Sigma 1 and Sigma 2 (Fig. 5). In this case the shear stresses are zero. Stresses Sigma 1 and Sigma 2 are provided form Stress2D software.

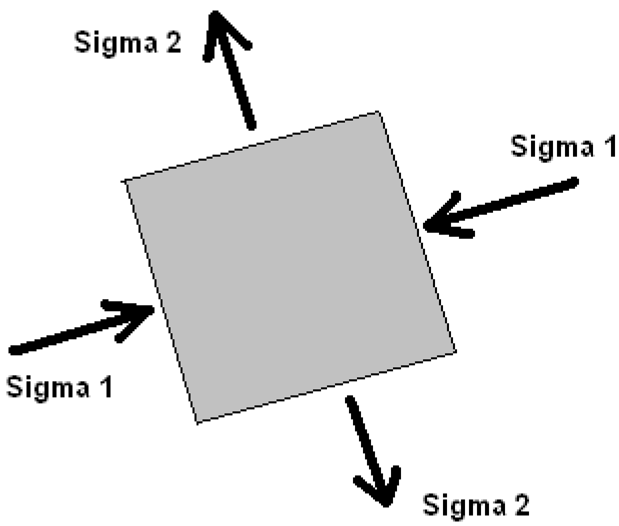

Fig. 5, Principal Stresses

In our example we can consider the principal stresses in a location just above the column on the left. (Fig. 6). At the central point stress Sigme 1 = 2,373 kPa (tension), and stress Sigma 2 = -6,226 kPa (compression). We have also shown stresses in two more points on each side, and above and below, to check is there any significant stress variation. We can observe that the above values are representative for that particular region, a 1x1m square. The angle of the principal stresses is 60 degrees. This can be observer from the orientation of the stresses (See Fig 6). Note that in this case the shear stresses (tau) are not present.

The unit stress prism, for that particular zone and stresses used in steel design are shown below in the figure (Fig 7). We may observe that the crack in the concrete may develop perpendicular to the direction of the tensile stresses. (Fig 7).

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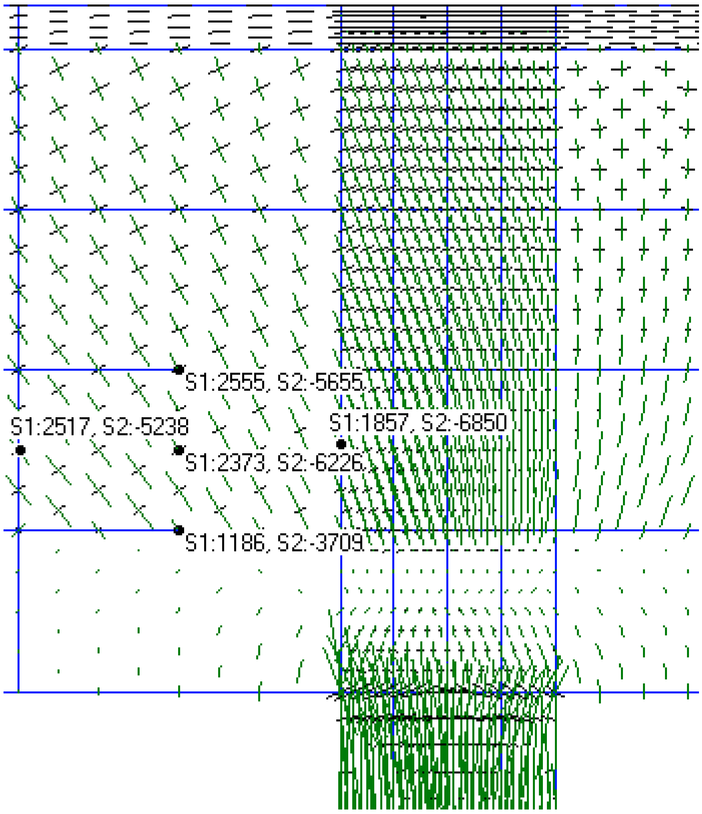

Fig. 6, Principal Stresses

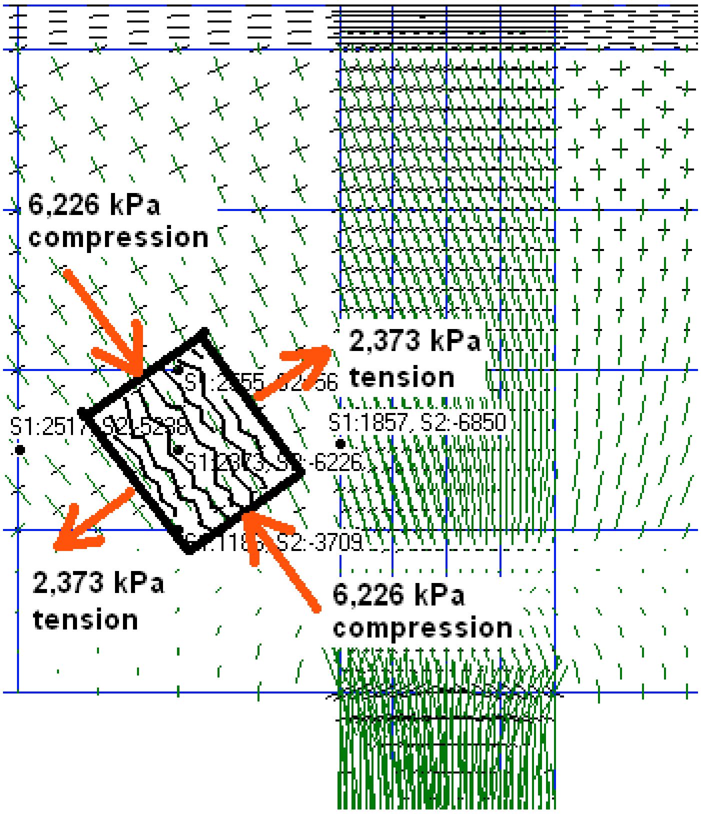

Fig. 7, Principal Stresses Used in Steel Design

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First of all we may observe that the compression stresses of 6,226 kPa is much lower than the compression strength of concrete, for instance 32 MPa (32,000 kPa). This is common in deep beam design.

The tensile stress of 2,373 kPa has to be taken but the reinforcement. If the section thickness is 0.190m, then the total tensile force over 1m height is:

N = 2,373 x 0.190 x 1.00 = 450 kN   tensile force over 1m length

If we ignore the tensile strength of the concrete, then we need to provide a sufficient number of horizontal reinforcing bars to limit the stress in the reinforcing steel to 1/4 of the ultimate steel strength. If the steel grade is 500 MPa, the maximum stress in the steel will be 125 MPa. This level of stress will provide adequate strength, and it will limit the cracks in the concrete.

Steel Area Required = 450 / 125,000 = 3,600 mm^2

Or, if we assume we will use bars N16 bars, on 150mm spacing or less than the allowable stress is 280 MPa:

Steel Area Required = 450 / 280,000 = 1,600 mm^2

However, we do not place the reinforcement on an angle, and therefore we have to split it in the vertical and horizontal direction, 4N16 in the vertical direction and 7N16 in the horizontal direction, over 1m length. In this manner we do not need to consider the shear stresses, and no additional reinforcement is needed to take the shear stresses. (Fig. 8)

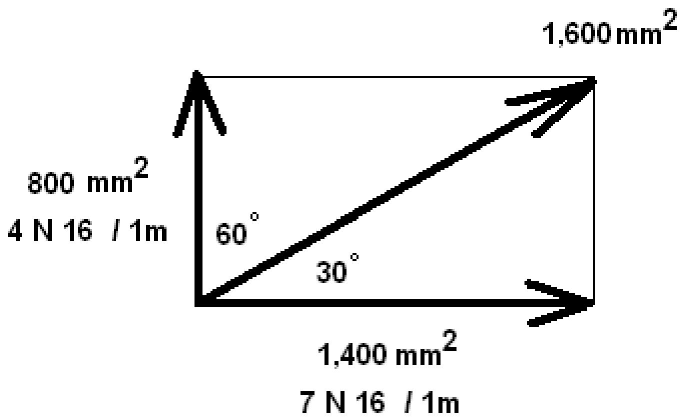

Fig 8, Reinforcement

The above procedure is the most accurate approach to design a deep beam.

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